1 Prerequisites

I will assume a willingness to work with *abstract measure spaces* and basic ideas of measurable functions, including Borel measures on separable metric spaces, $\sigma-$ algebras, integration, including integration of vector valued functions, L^p-spaces, almost sure convergence, convergence in measure/probability

From probability theory the concept and basic properties of *conditional expectations* will be assumed, as well as the basic Kolmogorovian approach to probabilty in terms of probability spaces, expectations as integrals etc. it would be helpful to know a definition and the basic properties of Brownian motion in R^m. Parts of the course will be more enjoyable given some basic ideas about Markov processes.

From calculus we will use *Frechet differentiation* of functions defined on open sets of Banach spaces with Banach space values, though mostly the spaces could be R^m. The first derivative at a point will be a bounded linear transformation of the spaces, the second often considered as a bounded bilinear transformation. It will be helpful to know the divergence theorem on $\mathbf{R}^n$ i.e. the integration by parts formula involving the divergence of a suitably regular vector field on $\mathbf{R}^n$.

There will not be time to do much about SDE on manifolds, but for what we do it would be useful to know the definition (preferably of a Banach manifold) and the concept of tangent spaces, vector fields, submanifolds and embeddings.

2 Tentative Schedule

The following is a rough guide rather than a strict timetable, and is likely to be modified a lot by the needs of the participants and the time available. In general there will not be time to give detailed proofs of results with good proofs in the literature.

2.1 Wednesday: lectures 1 & 2

Fourier transforms of Borel measures on Banach spaces, Gaussian measures and Abstract Wiener Spaces, Paley-Wiener integrals as divergences via the Gross- Cameron-Martin formula, Classical Wiener space and the Cameron-Martin space.

2.2 Wednesday: lecture 3

First look at Itô integrals, including Hilbert space valued integrals. First look at Itô's formula on Hilbert spaces, and stochastic differential equations. Stratonovich calculus. Piecewise linear approximation.

2.3 Thursday: lectures 4,5,6

Filtrations, stopping times, continuous martingale and semi-martingales, finite and quadratic variation. Optional stopping. Progressively measurable functions. Stochastic integration of suitable progressively measurable functions with respect to continuous semi-martingales, and corresponding Itô formulae. Fisk-Stratonovich integral. Brief introduction to SDE on manifolds and stochastic flows via SDE on the diffeomorphism group. Weak solutions and martingale problems.

2.4 Friday: lecture 7

Girsanov-Maruyama-Cameron-Martin theorem; Bismut type theorems.

2.5 Friday: lectures 8 and 9

Stochastic integrals as divergences on Wiener space, regularity of probability laws, Watanabe's distribution theory on Wiener space, Malliavin calculus.

3 Possibly useful references:

For a combination of the basic measure theory and some discrete martingale theory:
R.Schilling; *Measures, integrals, and martingales* Cambridge

There is the classic:
K.R.Parthasarathy; *Probability measures on metric spaces.* Probability and Mathematical Statistics, No. 3, Academic Press 1967

For some of the Gaussian Measure & Malliavin calculus there are the lecture notes
http://www.tjsullivan.org.uk/pdf/MA482-Stochastic-Analysis.pdf,
though rather imperfect. Some of the Gaussian measure material can be found in:

D. Stroock; *Probability Theory, an analytic view.* Cambridge University Press

For more about this see:
Y.Yamasaki; *Measures on infinite dimensional spaces* World Scientific Series in Pure Mathematic, 1985

or the very impressive:
Vladimir I. Bogachev;*Gaussian measures.* Mathematical Surveys and Monographs, 62. American Mathematical Society, Providence, RI, 1998. xii+433 pp. ISBN: 0-8218-1054-5

For calculus on Banach spaces and manifolds there is an Appendix to Elworthy below and Serge Lang's books, eg:
S.Lang' *Introduction to differentiable manifolds* Springer Universitext, ISBN 978-0-387-21772-7

For Malliavin Calculus there is Ikeda & Watanabe mentioned below and:
D.Nualart; *The Malliavin Calculus and related topics.* Springer.
(His notation and philosophy is rather different, partly because he uses L^2 when we use $L^{2,1}$, as Cameron-Martin space.)

Denis Bell's book may be found readable, useful, and close to parts of this mini-course:
Bell, Denis R; *The Malliavin calculus.* Reprint of the 1987 edition. Dover Publications, Inc., Mineola, NY, 2006. x+113 pp. ISBN: 0-486-44994-7 60-02

You might like bits of this if you can get hold of it:
Da Prato, Giuseppe. Introduction to stochastic analysis and Malliavin calculus. Second edition. Appunti. Scuola Normale Superiore di Pisa (Nuova Serie) [Lecture Notes. Scuola Normale Superiore di Pisa (New Series)], 7. Edizioni della Normale, Pisa, 2008. xvi+211 pp. ISBN: 978-88-7642-337-6; 88-7642-337-6

For continuous martingale calculus:
Revuz & Yor; *Continuous martingales and Brownian motion* Springer

N. Ikeda & S.Watanabe; *Stochastic differential equations and diffusion processes* North Holland (1989)
(The second edition is even better than the first, both have a good treatment of the martingale approach to diffusions. The second has Malliavin Calculus.)

The books by David Williams and Chris Rogers are pretty nice too and contain material about diffusions on manifolds:

Rogers, L. C. G.; Williams, David; *Diffusions, Markov processes, and martingales. Vol. 1. Foundations.* Reprint of the second (1994) edition. Cambridge Mathematical Library. Cambridge University Press, Cambridge, 2000. xx+386 pp. ISBN: 0-521-77594-9 60J60

Rogers, L. C. G.; Williams, David *Diffusions, Markov processes, and martingales. Vol. 2. It calculus.* Reprint of the second (1994) edition. Cambridge Mathematical Library. Cambridge University Press, Cambridge, 2000. xiv+480 pp. ISBN: 0-521-77593-0 60J60

K.D.Elworthy; *Stochastic Differential equations on manifolds* Cambridge University Press
(The book is old with first part rather outdated; but see the section on stochastic flows, the proof of the Girsanov-Maruyama-Cameron-Martin Theorem, & the appendix on manifolds)

For more about the diffeomorphism group approach to flows: see Carverhill-Elworthy for a treatment not using SDE on manifolds and Brzezniak-Elworthy for the extra precision using Banach space theory:
Carverhill, A. P.; Elworthy, K. D; *Flows of stochastic dynamical systems: the functional analytic approach.* Z. Wahrsch. Verw. Gebiete 65 (1983), no. 2, 245267.
Brzeżniak, Zdzislaw; Elworthy, K. D; *Stochastic flows of diffeomorphisms.* Stochastic analysis and applications (Powys, 1995), 107138, World Sci. Publ., River Edge, NJ, 1996.

For a general version of the Girsanov-Maruyama-Cameron-Martin Theorem: see the Appendix to

Elworthy, K. David; Le Jan, Yves; Li, Xue-Mei ; *The geometry of filtering.* Frontiers in Mathematics. Birkhuser Verlag, Basel, 2010. xii+169 pp. ISBN: 978-3-0346-0175-7.